Effects of chemical synapses on the enhancement of signal propagation in coupled neurons near the canard regime

Xiumin Li,* Jiang Wang,† and Wuhua Hu‡
School of Electrical Engineering and Automation, Tianjin University, Tianjin 300072, People's Republic of China
(Received 21 April 2007; revised manuscript received 27 July 2007; published 4 October 2007)

The response of three coupled FitzHugh-Nagumo neurons, under Gaussian white noise, to a subthreshold periodic signal is studied in this paper. By combining the canard dynamics, chemical coupling, and stochastic resonance together, the information transfer in this neural system is investigated. We find that chemical synaptic coupling is more efficient than the well-known linear coupling (gap junction) for local signal input, i.e., only one of the three neurons is subject to the periodic signal. This weak and local input is common in biological systems for the sake of low energy consumption.

DOI: 10.1103/PhysRevE.76.041902 PACS number(s): 87.19.La, 05.45.−a, 05.40.Ca

I. INTRODUCTION

Noise-induced complex dynamics in excitable neurons have attracted great interest in recent years. The random synaptic input from other neurons, random switching of ion channels, and the quasirandom release of neurotransmitter by synapses contribute to the randomicity in neurons [1]. Izhikevich [2] briefly introduced the influence of channel noise, conductance noise, membrane noise, and synaptic noise on the dynamics of neural systems. In contrast to the destructive role of noise, such as disordering or destabilizing the systems, noise can play important and constructive roles for the amplification of information transfer in some cases. Particularly, in the presence of noise, special attention has been paid to the complex behaviors of neurons that locate near the canard regime [3–8], where neurons are so sensitive to external signal that they can save energy consumption of biological systems in signal processing. Such neurons, as investigated in [3,4,7], possess two internal frequencies corresponding to the standard spiking and the small amplitude oscillations (canard orbits). For the former, it is the frequency of the regular spiking purely induced by appropriate noise, which is known as coherence resonance (CR). For the latter, the subthreshold oscillations are critical in the famous stochastic resonance (SR) phenomenon. SR describes the cooperative effect between a weak signal and noise in a nonlinear system, leading to an enhanced response to the periodic force [9].

Recently, Ullner *et al.* gave detailed descriptions of several new noise-induced phenomenon in the FitzHugh-Nagumo (FHN) neuron in [1]. They showed that optimal amplitude of high-frequency driving enhances the response of an excitable system to a low-frequency signal [10]. They also investigated the Canard-enhanced SR [4], the effect of noise-induced signal processing in systems with complex attractors [11], and a new noise-induced phase transition from a self-sustained oscillatory regime to an excitable behavior [12]. In [13], Zhou *et al.* have demonstrated the effect of CR in a heterogeneous array of coupled FHN neurons. They find that both the decrease of spatial correlation of the noise and the inhomogeneity in the parameters of the array can enhance the coherence.

However, most of the relevant studies only considered the single neuron [4,6,14] or neurons with linear electrical coupling (gap junctions) [5,13,15,16] and omitted another important case—chemical (nonlinear) coupling. As investigated in [17], a substantial increase in the CR of chemical coupled Morris-Lecar models can be observed, in comparison with the (linear) electrical coupled ones.

Considering these, based on the canard dynamics in chemical coupled neurons [18], we make comparisons of the response to external periodic signal between chemical coupled and electrical coupled neurons, which locate near the canard regime and are subject to white noise environment. The contents of this paper are arranged as follows. In Sec. II, brief introductions of the FHN model, the two coupling cases, and the simulation are given, and then a comparison between the two kinds of coupling are made for information transfer. Finally, conclusions and discussions are made in Sec. III.

II. CHEMICAL SYNAPSES VERSUS GAP JUNCTIONS

A. Neuron model and coupling description

We consider a model of three bidirectional coupled FHN models [19] described by

$$\varepsilon \dot{V}_i = V_i - \tfrac{1}{3}V_i^3 - W_i + I^{\mathrm{app}} - I_i^{\mathrm{syn}},$$

$$\dot{W}_i = V_i + a - b_i W_i + B_i \cos(\omega t) + A\xi_i(t), \qquad (1)$$

where $i=1,2,3$ index the neurons, a, b_i, and ε are dimensionless parameters with $\varepsilon \ll 1$ that make membrane potential, V_i, a fast variable and recovery variable, W_i, a slow variable. In this section, $b_1=b_2=b_3=b$ and ξ_i are independent Gaussian white noises with zero mean and intensity A for each element. $B_i \cos(\omega t)$ is the forcing periodic signal. I^{app} and I_i^{syn} are, respectively, the external applied current and the synaptic current through neuron i.

For the electrical coupling,

*free_xmin@yahoo.com.cn
†jiangwang@tju.edu.cn
‡haitun_ahua@yahoo.com.cn

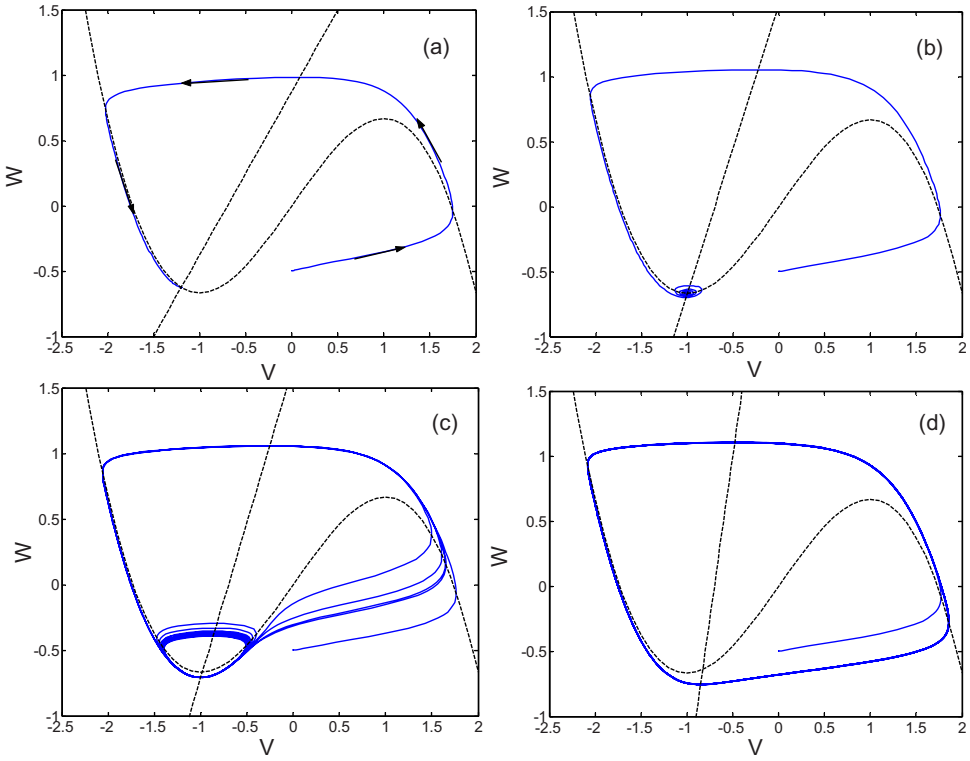

FIG. 1. (Color online) Phase portraits (blue solid line) of the single FHN neuron without noise and external input, where $\varepsilon=0.08$, $I^{\mathrm{app}}=0$, $a=0.7$, and b is the critical parameter with the threshold $b_0=0.45$. Two black dashed lines are V and W null clines, respectively. (a) $b>b_0$, (b) $b=b_0$, (c) $b\approx b_0$, (d) $b<b_0$.

$$I_i^{\mathrm{syn}} = \sum_{j\in \mathrm{neigh}(i)} g_{\mathrm{syn}}(V_i - V_j), \tag{2}$$

where g_{syn} is the conductance of synaptic channel.

For the chemical coupling [20,21],

$$I_i^{\mathrm{syn}} = \sum_{j\in \mathrm{neigh}(i)} g_{\mathrm{syn}} s_j (V_i - V_{\mathrm{syn}}), \tag{3}$$

where g_{syn} is the synaptic coupling strength and V_{syn} is the synaptic reversal potential that determines the type of synapse. For the excitatory synapse considered in this paper, $V_{\mathrm{syn}}=0$. The dynamics of the synapse variable s_j is governed by V_j. s_j is defined by

$$\dot{s}_j = \alpha(V_j)(1-s_j)/\varepsilon - s_j/\tau_{\mathrm{syn}},$$

$$\alpha(V_j) = \frac{\alpha_0}{1+\exp(-V_j/V_{\mathrm{shp}})}, \tag{4}$$

where synaptic decay rate τ_{syn} is equal to $1/\delta$. The synaptic recovery function $\alpha(V_j)$ can be taken as the Heaviside function. When the neuron is in silent state ($V<0$), s is slowly decreasing and the first equation of (4) can be taken as $\dot{s}_j=-s_j/\tau_{\mathrm{syn}}$; while in the other case, s jumps fast to 1 and acts on the postsynaptic cells. Note that in this coupling case neuron is coupled only when its presynaptic neuron is active, which is quite different from the continuous connection between electrical coupled neurons.

In this model, b is one of the critical parameters that can significantly influence the dynamics of the system (see Fig. 1). For a single neuron free from noise, Andronov-Hopf bifurcation happens at $b_0=0.45$. If $b>b_0$, the neuron would be excitable and corresponds to the rest state; if $b<b_0$, the system would possess a stable periodic solution generating periodic spikes. Between these two states, there also exists an intermediate behavior, known as canard explosion [21]. In a small vicinity of $b=b_0$, there are small oscillations near the unstable fix point before the sudden elevation of the oscillatory amplitude. This canard regime tends to zero as the parameter $\varepsilon\to 0$. Here we take $\varepsilon=0.08$ as used in [22], and in this case the canard regime exists for $b\in[0.425,0.45]$. This regime is very sensitive to external perturbations and thus plays a significant role in the signal propagation, which will be further discussed below.

B. Introduction of the simulations

Stochastic resonance describes the optimal synchronization of the neuron output with the weak external input signal due to intermediate noise intensity. It is closely related to the information transfer in neural systems. In this paper we first study SR in three coupled neurons with local stimulus, that is, only one element is subject to external periodic signal. The parameters of input periodic signal are taken as $B_1=0.05$, $B_2=0$, $B_3=0$, and $\omega=0.3$ so that there are no spiking for all the neurons in the absence of noise. Note that the value of ω is much smaller than the two internal frequencies of neurons.

Figure 2 shows the optimal response of neurons to the local input signal with intermediate intensity of noise. And we can see that, for large enough coupling strength g_{syn}, time traces of electrical coupled neurons are basically identical. While in the chemical coupling case, there exists a slight delay between spikes and the subthreshold oscillations are different from each other (see Fig. 2). Therefore, we only examine the response of the second neuron to external input instead of the mean field. That is $V_i=V_2$ in the calculation of

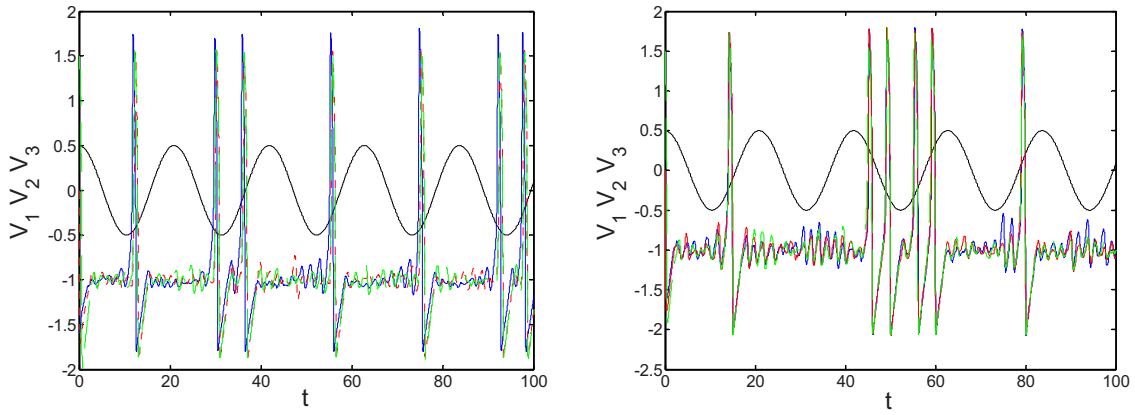

FIG. 2. (Color online) Time series of V_i $(i=1,2,3)$ and the input signal (black line, the amplitude is 10 times higher than that in the model). Left-hand side, chemical coupling $b=0.45$, $g_{\mathrm{syn}}=0.15$, $A=0.015$; right-hand side, electrical coupling $b=0.45$, $g_{\mathrm{syn}}=0.1$, $A=0.035$.

Fourier coefficient Q, which is used to evaluate the response of output frequency to the input frequency. The definition of Q [23] is

$$Q_{\sin} = \frac{\omega}{2\pi n}\int_0^{2\pi n/\omega} 2V_i(t)\sin(\omega t)dt,$$

$$Q_{\cos} = \frac{\omega}{2\pi n}\int_0^{2\pi n/\omega} 2V_i(t)\cos(\omega t)dt,$$

$$Q = \sqrt{Q_{\sin}^2 + Q_{\cos}^2}, \tag{5}$$

where n is the number of periods $2\pi/\omega$ covered by the integration time. The quantity Q measures the component from

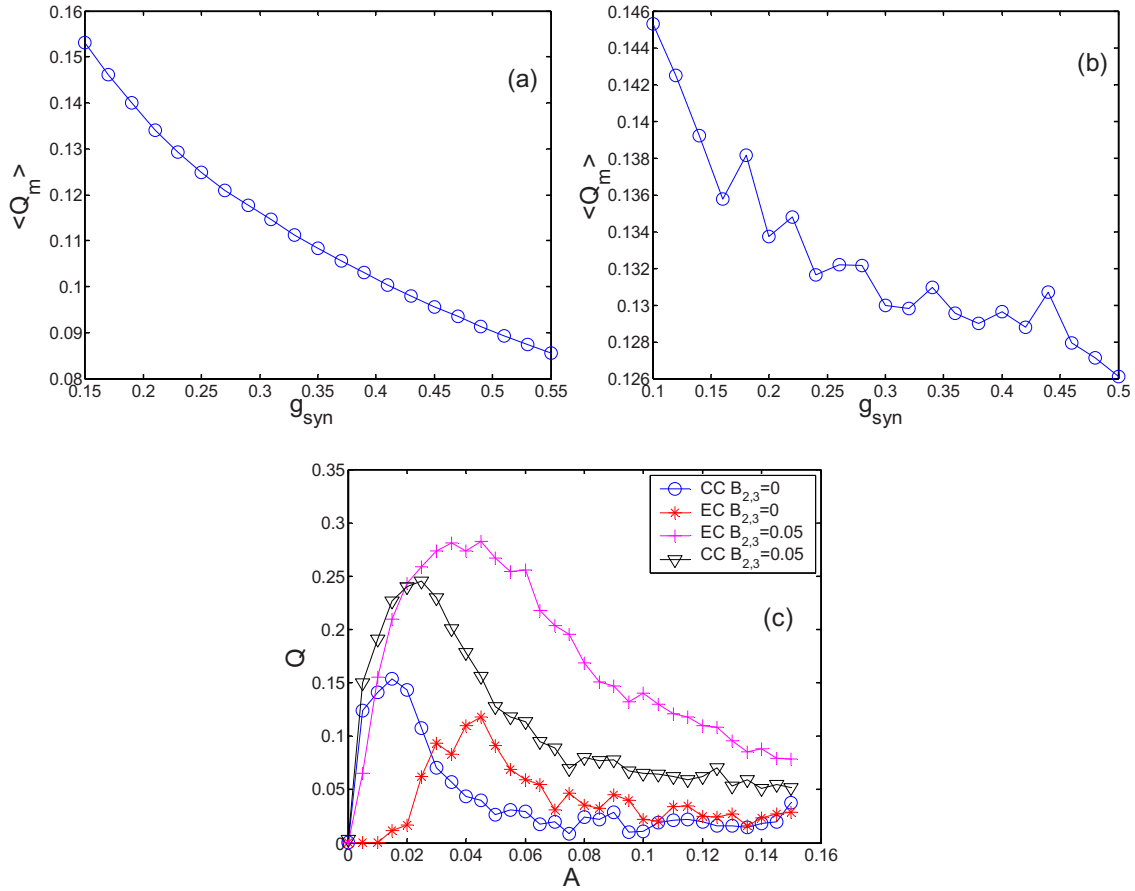

FIG. 3. (Color online) (a) and (b) The average of Q_m over 10 different noise realizations for different g_{syn} in two respective kinds of coupled neurons, with $b=0.45$; (c) signal processing at the input signal versus the noise intensity for the coupled system in different cases, where $b=0.45$, $B_1=0.05$; CC, $g_{\mathrm{syn}}=0.15$; EC, $g_{\mathrm{syn}}=0.1$.

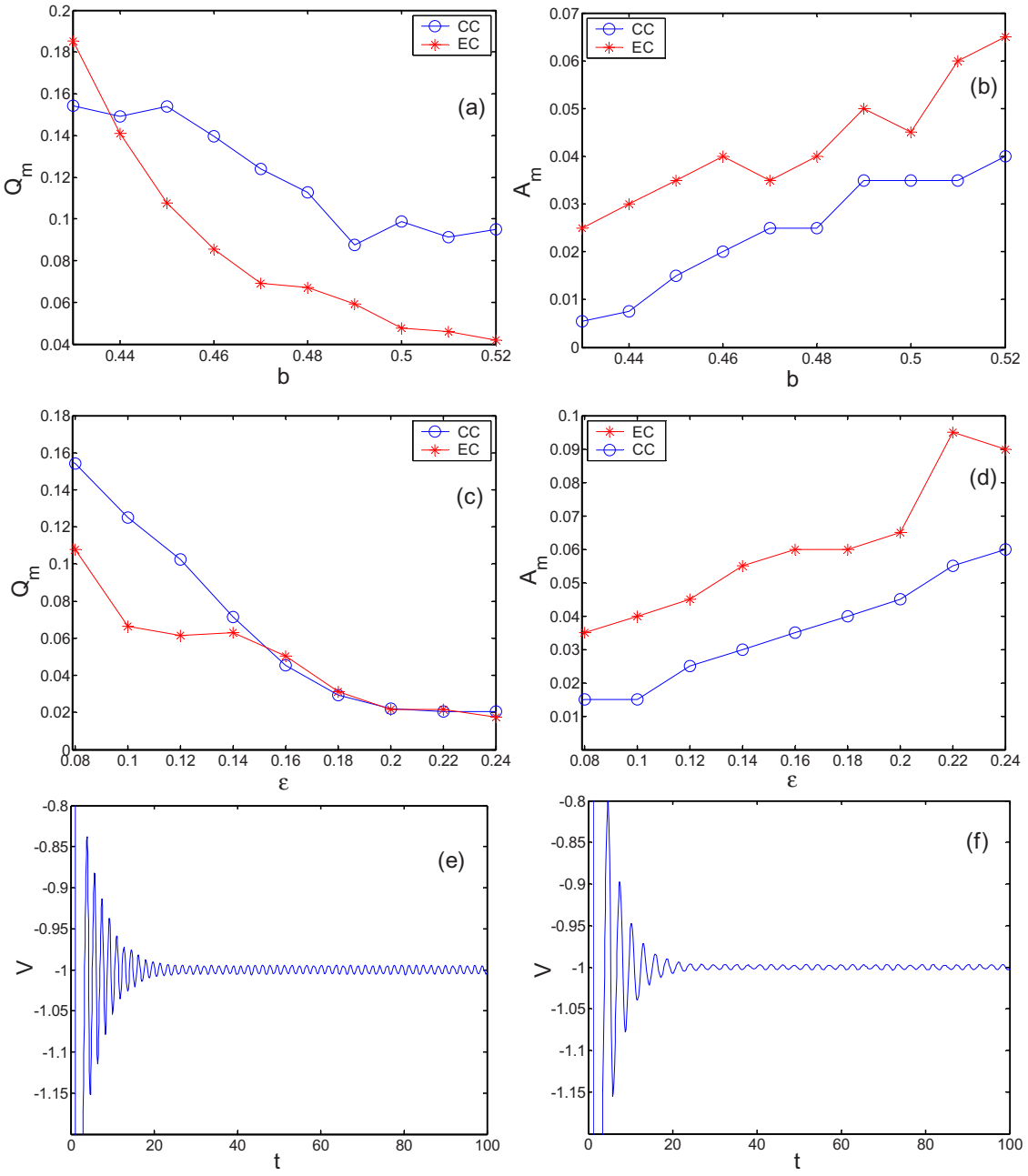

FIG. 4. (Color online) (a)–(d) The maxim of Q (Q_m) and the corresponding noise intensity A_m for different parameters b and ε in two respective coupling cases, where CC, g_{syn}=0.15; EC, g_{syn}=0.1. (e) and (f) Time series of the membrane potential V in a single neuron with different ε: (e) ε=0.08, (f) ε=0.2.

the Fourier spectrum at the signal frequency ω. The maximum of Q shows the best phase synchronization between input signal and output firing. Also, as information in neuron systems is carried through large spikes instead of subthreshold oscillations, we are more interested in the frequency of spikes. So following [4], we set the threshold V_s=0 in the calculation of Q. If $V<V_s$, V is replaced by the value of the fixed point V_f (here V_f=−1); otherwise, V remains the same.

The parameters used in this paper are a=0.7, ε=0.08, V_{syn}=0, α_0=2, V_{shp}=0.05, δ=1.2, I^{app}=0. The rest parameters are given in each case. And the numerical integrations of the system are done by the explicit Euler-Maruyama algorithm [24], with a time step 0.005.

C. Results and discussions

We study the differences between the chemical coupling and the electrical coupling for SR when the neurons locate near the bifurcation point b=0.45. To investigate the influence of coupling strength, the maximums of Q (Q_m) are calculated at the corresponding optimal noise intensities for different values of g_{syn} in two respective coupling cases [see Figs. 3(a) and 3(b)]. $\langle Q_m \rangle$ is the average of Q_m over 10 different noise realizations. g_{syn}=0.15 in (a) and g_{syn}=0.1 in (b) are the smallest values for neurons to fire synchronously. The fluctuations in Fig. 3(b) are caused by the sensitivity to noise of the electrical coupled neurons, due to the great de-

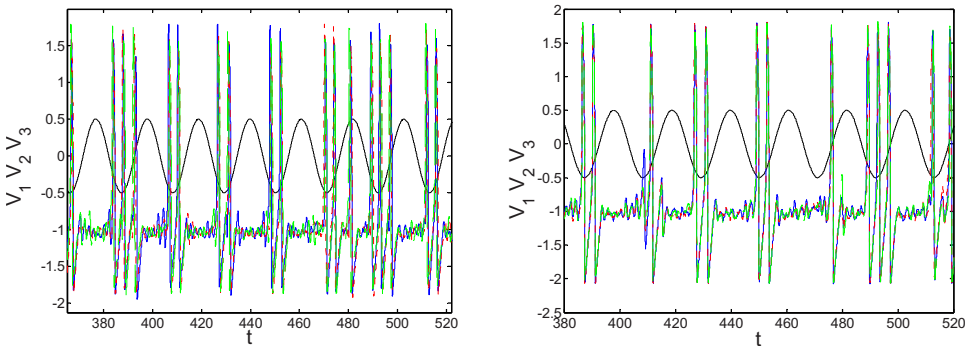

FIG. 5. (Color online) Time series of V_i $(i=1,2,3)$ and input signal (black line, the amplitude is 10 times higher than that in the model). Left-hand side, chemical coupling $b=0.45$, $g_{\mathrm{syn}}=0.15$, $A=0.025$, $B_{1,2,3}=0.05$. Right-hand side, electrical coupling $b=0.45$, $g_{\mathrm{syn}}=0.1$, $A=0.035$, $B_{1,2,3}=0.05$.

pendence on noise to make fires in this coupling case. It is obvious that both too weak and too strong couplings can decrease the ability of signal processing in each case. For chemical synapses, too strong coupling distorts the wave form with random amplitude of the membrane potential and thus decreases the value of Q. For the electrical coupling, strong coupling means strong synchronization between cells, which may suppress the subthreshold oscillations and make the system need large noise to fire. Considering these, we choose an appropriate coupling strength in this paper, say $g_{\mathrm{syn}}=0.15$ for the chemical coupling case and $g_{\mathrm{syn}}=0.1$ for the electrical coupling case.

As we can see, for local stimulus $B_1=0.05$, $B_2=0$, $B_3=0$, chemical coupling is more efficient than electrical coupling for signal processing [Fig. 3(c)]. As discussed in [17], chemical synapses only act while the presynaptic neuron is spiking, whereas electrical coupling connects neurons at all times (Fig. 2). Chemical coupling enables small oscillatory neurons to be free from each other and gives more opportunities for them to fire. Once one spikes, it will stir the others to spike synchronously. While for the electrical coupling, strong synchronizations between subthreshold oscillatory neurons result in the decrease of oscillatory amplitude and thus the increase of the threshold for firing. Therefore, chemical coupled neurons can make better explorations of the internal sensitive dynamics and need smaller noise to complete signal processing than electrical coupled ones.

Figure 4 shows the influence of subthreshold oscillations (canards) on the signal processing by changing two parameters b and ε. With the increase of b, the excitation threshold increases and the system gradually escapes from the canard regime. These lead to the decrease of SR in both of the two coupling cases [see Figs. 4(a) and 4(b)]. With the same excitation threshold ($b=0.45$), the weakening of subthreshold oscillations which is induced by the increase of ε can also lead to the decline of SR [see Figs. 4(c)–4(f)]. When $\varepsilon>0.16$, the superiority of chemical coupling over electrical coupling disappears. From this phenomenon, we can learn that subthreshold oscillations are very important for the firing of large spikes.

Additionally, we investigate the global stimulus $B_{1,2,3}=0.05$, where each neuron is forced by the external signal. Here the chemical coupled system is not as efficient as the electrical coupled one for SR [Fig. 3(c)]. In this case, neurons are more active and can fire easily, induced by the external signal and noise. The continuous connection in electrical coupled neurons leads to high synchronization and can make better control of the firing rate than the selective connection in chemical coupled neurons (see Fig. 5).

However, the global input is not common in real neural systems. In fact, local input is a more ubiquitous case rather than a more restricted case. In neural systems with a large amount of cells, it is unnecessary and impossible to add external signals to all the involved individuals. Only weak and local input is reasonable and guarantees the low energy consumption in large neural networks. This may be relevant to the fact that chemical coupling is more universal in mammals than electrical coupling.

III. CONCLUSION

In this paper, we make comparisons of the response to external signal between chemical coupled and electrical coupled noisy neurons. In the global input case, the continuous synchronizations of the electrical coupled neurons can control the frequent firing rate and thus behave better (SR) than chemical coupled ones. While in the more common case, i.e., local input case, chemical coupling is more effective for this weak signal propagation due to its selective coupling. This is very important in practical systems. As in neural systems with a large amount of cells, only weak and local input is reasonable and guarantees the low energy consumption in signal processing. This may be relevant to the fact that chemical coupling is more universal in mammals than electrical coupling.

It should be noted that canard dynamics, which had been discussed in [21,25], plays a critical role to signal processing. The number of subthreshold oscillations between two closest large spikes has close relationships to the firing rate, which carries the information during signal propagation. We will further this study and extend it to larger size of networks with different topological connections.

ACKNOWLEDGMENTS

The authors gratefully acknowledge Gang Zhao for the helpful discussion. This work is supported by the NSFC Grant No. 50537030.

[1] E. Ullner, Dissertation, Institute of Physics, Potsdam University, 2004.
[2] E. M. Izhikevich, *Dynamical Systems in Neuroscience* (MIT Press, Cambridge, MA, 2005).
[3] M. Perc and M. Marhl, Phys. Rev. E **71**, 026229 (2005).
[4] E. I. Volkov, E. Ullner, A. A. Zaikin, and J. Kurths, Phys. Rev. E **68**, 026214 (2003).
[5] G. Zhao, Z. Hou, and H. Xin, Chaos **16**, 043107 (2006).
[6] M. A. Zaks, X. Sailer, L. Schimansky-Geier, and A. B. Neiman, Chaos **15**, 026117 (2005).
[7] V. A. Makarov, V. I. Nekorkin, and M. G. Velarde, Phys. Rev. Lett. **86**, 3431 (2001).
[8] A. Shishkin and D. Postnov, *Physics and Control, Proceedings of the International Conference 2*, PHYSCON-2003, pp. 649–653.
[9] T. Wellens, V. Shatokhin, and A. Buchleitner, Rep. Prog. Phys. **67**, 45 (2004).
[10] E. Ullner *et al.*, Phys. Lett. A **312**, 348 (2003).
[11] E. I. Volkov, E. Ullner, A. A. Zaikin, and J. Kurths, Phys. Rev. E **68**, 061112 (2003).
[12] E. Ullner, A. Zaikin, J. García-Ojalvo, and J. Kurths, Phys. Rev. Lett. **91**, 180601 (2003).
[13] C. Zhou, J. Kurths, and B. Hu, Phys. Rev. Lett. **87**, 098101 (2001).
[14] P. L. Gong and J. X. Xu, Phys. Rev. E **63**, 031906 (2001).
[15] R. Toral, C. R. Mirasso, and J. D. Gunton, Europhys. Lett. **61**, 162 (2003).
[16] J. M. Casado and J. P. Baltanás, Phys. Rev. E **68**, 061917 (2003).
[17] P. Balenzuela and J. Garcia-Ojalvo, Phys. Rev. E **72**, 021901 (2005).
[18] J. Wang, X. Li, and W. Hu (unpublished).
[19] R. FitzHugh, Biophys. J. **1**, 445 (1961).
[20] J. Drover, J. Rubin, J. Su, and B. Ermentrout, SIAM J. Appl. Math. **65**, 69 (2004).
[21] M. Wechselberger, SIAM J. Appl. Dyn. Syst. **4**, 101 (2005).
[22] J. Cronin, *Mathematical Aspects of Hodgkin-Huxley Neural Theory* (Cambridge University Press, Cambridge, 1987).
[23] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[24] Desmond J. Higham, SIAM Rev. **43**, 525 (2001).
[25] P. Szmolyan and M. Wechselberger, J. Differ. Equations **177**, 419 (2001).